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SHORT NOTES

The pole of the Mohr diagram

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Abstract—The pole of a Mohr diagram, for the two-dimensional case, is a unique point on the Mohr circle which permits any point on the Mohr circle to be related to the direction in the physical plane associated with that point. A Mohr diagram can be constructed for any second rank tensor. To illustrate the simplicity of this geometrical construction two examples of the use of the pole are presented, one for the strain tensor and the other for the stress tensor.

IN TWO recent papers there has been reference to the use of the pole on a Mohr diagram (Mandl & Shippam 1981, Cutler & Elliott 1983). In one case the construction was for stress (Mandl & Shippam 1981) and in the other for finite strain (Cutler & Elliott 1983). A third recent paper on the Mohr circle construction did not use the pole but this could usefully have been included (Means 1982). In the first two papers, a brief description is given of the properties of the pole. However, the use of the pole is, apparently, not commonly known to structural geologists except, perhaps, those with a soil mechanics background.

The use of the pole permits values of strain (or stress) to be related to their orientations in physical space by a simple geometrical technique rather than by more complicated numerical calculations. It is often surprising how such simple techniques have remained in the literature for years without their being utilised by certain groups of scientists (cf. Treagus 1981).

I illustrate the use of the pole in a Mohr diagram by two simple examples, one for strain and the other for stress.

THE POLE

For simplicity the description in the following two paragraphs is in terms of Cauchy's finite strain tensor but the description is valid for any second rank tensor. In two dimensions a Mohr circle describes a homogeneous state of strain. In the Mohr diagram all directions are measured relative to the directions of the principal strains. If the orientation in physical space is known for a line of specified strain, represented by a point on the Mohr circle, the strain point, then the orientations of lines with other values of strain, for example the principal strains, may be calculated numerically or may simply be determined geometrically using the pole.

The pole is a unique point on the Mohr circle for a given state of homogeneous strain and orientation in space of the principal strains with respect to a fixed reference frame external to the strain markers. If either the state of strain or the orientation in physical space of the principal strains changes then the position of the pole in the Mohr diagram also changes.

The pole in a Mohr diagram for strain

Cutler & Elliott (1983) have defined the pole as "the point of intersection, on the Mohr circle, of all lines which go through strain points and are parallel to the material lines which those points represent". The relationship between the physical plane and the Mohr diagram is shown in Fig. 1(a).

The pole in a Mohr diagram for stress

It is apparent from the examples given in the literature that the pole in a Mohr diagram for stress has been defined in two different ways. Ford (Ford & Alexander 1977) stated that the line from a stress point to the pole is parallel to the trace of the physical plane on which the stress acts (Fig. 1b). Mandl & Shippam (1981) concurred with this definition. Cutler & Elliott (1983) briefly defined the pole in a Mohr diagram for stress but presented no illustration of it. They stated that the line from the stress point to the pole is parallel to the normal to the plane on which the stress acts (Fig. 1c). Goodman (1980) presented a construction which uses this latter definition of the pole. From Fig. 1 it can be seen that the two definitions give poles 180° apart. Consistent use of either definition in a problem will lead to correct results.

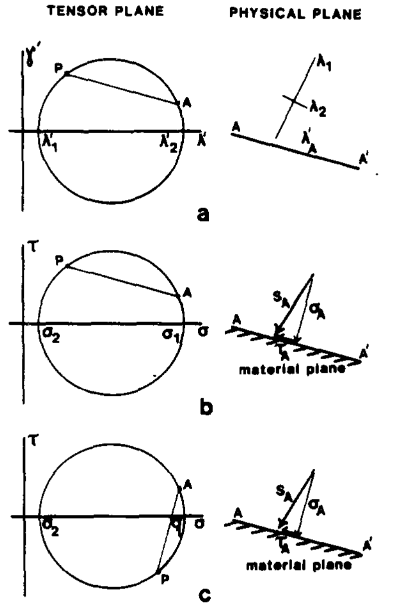

Fig. 1. The relationship between the physical plane and the tensor plane using the pole. (a) Strain. In the strain plane point A on the Mohr circle represents the strain, λ_A, associated with the line AA' in the physical plane. A chord from A in the Mohr diagram, drawn parallel to the line AA' in the physical plane, intersects the Mohr circle at P, the pole. (b) Stress. This construction follows the definition of the pole given by Ford & Alexander (1977) and Mandl & Shippam (1981). In the stress plane, point A represents the stress, S_A, acting on a material plane whose trace, in the physical plane, is AA'. The normal, σ_A, and shear, τ_A, stress components are also shown. A chord from A on the Mohr circle is drawn parallel to AA' in the physical plane and intersects the circle at P, the pole, (c) Stress. This construction follows the definition of the pole given by Goodman (1980) and Cutler & Elliott (1983). In the stress plane, point A represents the stress, S_A, acting on a material plane, AA', in the physical plane. Normal, σ_A, and shear, τ_A, stress components are also shown. In the stress plane, a chord is drawn from A parallel to the normal to the plane AA' and intersects the circle at P, the pole.

If the Cutler & Elliott (1983) definition is taken then there is a degree of correspondence between the cases for strain and stress in that the direction of finite longitudinal strain in physical space is parallel to the chord from the pole to the strain point and the normal component of the stress vector is parallel to the chord from the pole to the stress point. In spite of the fact that the construction of Mohr (1914) has historical precedence it is suggested that the form proposed by Cutler & Elliott (1983) be adopted as standard. A suitable definition of the pole in a Mohr diagram for stress is, therefore: the pole is the unique point of intersection, on the Mohr circle, of all chords drawn through stress points and parallel to the normals to the physical planes on which these stresses act.

EXAMPLES

Strain

This example is based on the problem of determining the strains of two bilaterally symmetrical fossils given by Ramsay (1967, p. 236). Following Ramsay's method a Mohr diagram can be constructed where two such fossils have been homogeneously deformed. The physical plane with two deformed brachiopods and the resultant Mohr diagram are shown in Fig. 2. Considering fossil A, the strain point on the Mohr circle corresponding to the strain in a direction parallel to the hinge line is h_A and that of the line of symmetry or median line is m_A. A chord drawn from h_A parallel to the hinge line in the physical plane intersects the circle at P, the pole. The direction in the physical plane associated with any strain point on the Mohr circle may now be easily determined. For example, the direction of the maximum extension in the plane is obtained by drawing a chord from the pole, P, to the strain point, λ_1', reciprocal of the maximum principal quadratic elongation. This chord is parallel to the direction of λ_1, maximum principal quadratic elongation, on the physical plane.

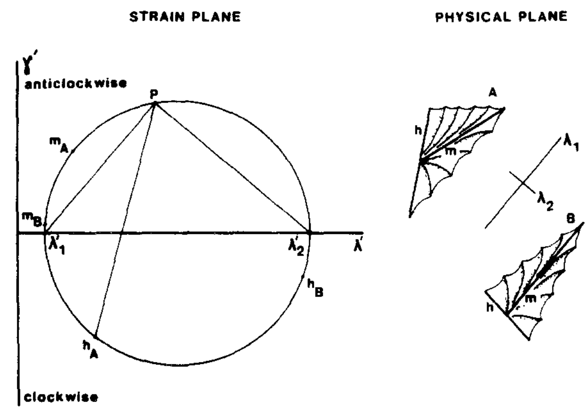

Fig. 2. The use of the pole in a strain problem. In the strain plane various points are marked on the Mohr circle and correspond to the strains, parallel to the directions on the physical plane, with the same symbols. In the physical plane two brachiopods, A and B, deformed by a homogeneous strain are shown. h refers to the direction of the hinge line and m to the direction of the median line; subscripts relate to the individual fossils. Having constructed the Mohr diagram following the method of Ramsay (1967, p. 236) a chord is drawn from strain point, h_A (or any other known strain point), parallel to the line, h_A, in the physical plane to intersect the Mohr circle at P, the pole. The chords from the pole, P, to the λ_1' and λ_2' points on the Mohr circle are shown and their orientations have been transferred to the physical plane. For clarity chords from the pole to the other marked strain points have been omitted but it is clear that they are parallel to the direction in the physical plane associated with these strains.

Stress

Figure 3 shows the trace of a fault on a physical plane and a corresponding Mohr diagram for stresses in this situation. The stress point, A, on the Mohr circle corresponds to the normal and shear stresses on the fault. From A a chord is drawn, parallel to the normal to the fault in the physical plane, to intersect the circle at P, the pole. To determine, say, the direction of the maximum principal compressive stress, a chord is drawn from the pole, P, to the stress point, σ_1, the maximum principal compressive stress. This chord is normal to the trace of the plane on which σ_1 acts and the direction of σ_1 is normal to this line (Fig. 3).

CONCLUSIONS

The pole is a unique point on a Mohr circle and its use

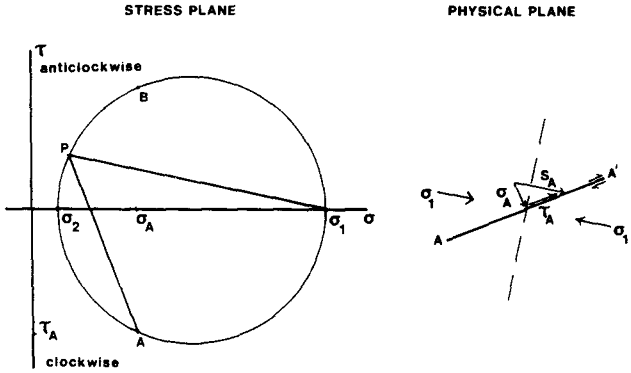

Fig. 3. The use of the pole in a stress problem. In the physical plane a fault whose trace is AA' is acted on by a stress, S_A, whose normal, σ_A, and shear, τ_A, stress components are shown. In the stress plane this stress is represented by point A on the Mohr circle. A chord from A, drawn parallel to the normal to the fault AA', in the physical plane intersects the Mohr circle at P, the pole. To determine, say, the direction of the maximum principal compressive stress, σ_1, a chord is drawn from P to σ_1 on the Mohr circle. In the physical plane the dashed line is normal to chord $P\sigma_1$, and is the trace of the plane on which σ_1 acts. The σ_1 direction is marked. Stress point, B, is associated with the conjugate fault to AA' and the trace of this conjugate fault can easily be determed as being normal to the chord, BP.

in a Mohr diagram enables points on the Mohr circle, representing values of strain or stress, to be related to the directions in the physical plane with which these values are associated. The technique is a simple geometrical one and obviates the need for long numerical calculations to derive the same information. The examples presented here have been deliberately kept simple so that the use of the pole can be appreciated easily.

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